Phys 410
Fall 2013
Lecture #27 Summary
5 December, 2013

We continued our discussion of Special Relativity. Einstein made two postulates:

1) If S is an inertial reference frame and if a second frame S' moves with constant velocity relative to S, then S' is also an inertial reference frame.
2) The speed of light (in vacuum) has the same value c in every direction in all inertial reference frames.

We examined the relativity of time by considering two reference frames, one with railroad tracks at rest (S), and the other (S') on a train moving down the tracks at a high rate of speed (V). Consider a light-clock on the train (frame S') that sends a brief flash of light from the floor to the ceiling, where it bounces off of a mirror, and then back to a detector that is co-located with the source on the floor. The time interval for the round trip of the light beam is $\Delta t' = 2h/c$, where h is the height of the train and c is the speed of light, as measured in S'. An observer (or really a set of observers) in S see the light follow a triangular trajectory as the train wizzes by. From the geometry of the experiment, and the second postulate, those observers attribute a time interval for the "round trip" of $\Delta t = \gamma \Delta t'$, where $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = V/c$. Since $\gamma > 1$ the two observers do not agree on how much time elapsed on the light-clock! This shows that the Galilean idea of universal time for all inertial observers is incorrect. In addition, because γ diverges as $V \to c$, it says that there is a speed limit for inertial reference frames: $V < c$. (This also means that we cannot address the question of what the world looks like from the reference frame of a photon travelling at the speed of light.)

The first postulate implies the equality of all inertial reference frames, so why is the result $\Delta t = \gamma \Delta t'$ asymmetric between the two inertial reference frames? The difference arises because the time interval was measured at a single fixed location in S' while it was measured at two distinct locations in S. The measurement of a time interval at a fixed location in an inertial reference frame is called the 'proper time interval' and is denoted Δt_0. Measurements of these two events taken from any other inertial reference frame moving with respect to this one will result in a dilated time interval measurement $\Delta t = \gamma \Delta t_0$.

We next considered the measurement of length (ℓ) in two different inertial reference frames. This led to the length contraction result: $\ell = \ell_0/\gamma$, where ℓ_0 is the 'proper length' of an object, namely the length when the object is at rest in your reference frame.

We discussed the inadequacy of the Galilean transformation of coordinates between two different inertial reference frames S and S'. For example the translation of x-coordinates between two reference frames moving at speed V in the x-direction is $x' = x - Vt$. But the two observers cannot even agree on time, so this equation is of little use. We derived the relativistic version of this transformation between S and S' moving at speed V in the x-direction, making use of length contraction to arrive at the Lorentz transformation:

$$x' = \gamma(x - Vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - xV/c^2)$$

Note that these equations reduce to the Galilean version in the limit $\frac{V}{c} \ll 1$. These equations show how a single event (in space-time) is described in two different reference frames that are moving a speed V relative to each other in the x-direction.

We applied the Lorentz transformation to the apparent paradox of a 100-cm-long relativistic stuffed animal moving across a table (at $\frac{V}{c} = 0.6$) that has two knives falling simultaneously at a distance 100 cm apart. From the perspective of reference frame S at rest with respect to the table the stuffed animal will be Lorentz contracted and easily fit between the falling knives. From the stuffed animal's perspective in frame S' the two knives appear to be only 80 cm apart, meaning that it will surely be cut in two by the two falling knives. The resolution of this paradox is a careful evaluation of the locations in space and time of the two falling knives using the Lorentz transformation. We found that in the stuffed animal's frame of reference the first knife just misses its tail, but the second knife falls 2.5 ns before the first and at a location of 125 cm in its frame, missing the stuffed animal altogether. Hence the paradox is resolved. However the results seem unsettling because the events that were simultaneous in S are no longer simultaneous in S'. In addition, the two knives appear to be too far apart in S'. These issues arise because we are used to dealing with situations where information travels much faster (at the speed of light!) compared to the motions of the objects of interest, and the distances covered in time Δt are very small compared to $c\Delta t$. Hence we can get a 'global' view of the system and ascribe a single universal time coordinate to the motion. This is no longer the case when objects are moving at speeds approaching light speed. It takes significant time for information to travel between two spatially separated points, and these delays must be incorporated into our description of the motion.

Finally we deduced the relativistic velocity addition formula from the differential form of the (linear) Lorentz transformation. Velocities of objects measured in frames S' and S moving at relative speed V in the x-direction are related as $v_x' = \frac{v_x - V}{1 - Vv_x/c^2}$, and $v_y' = \frac{v_y}{\gamma(1 - Vv_x/c^2)}$, $v_z' = \frac{v_z}{\gamma(1 - Vv_x/c^2)}$. For example if a spaceship is approaching earth at a speed of $\frac{V}{c} = 0.8$ and launches a light beam towards us ($v_x' = c$) then we measure the speed of that light as not $1.8\,c$, but as $v_x = c$, in accordance with the second postulate of relativity. The velocity addition formulas thus enforce the speed limit of the universe!